ON A COMPLETE STATISTICAL CHARACTERIZATION OF TURBULENCE

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1. Introduction

The common picture of fully developed local isotropic turbulence is that the velocity field is driven by external fields on large scales. By this, driving energy is fed into the system at scales larger than the integral length $L_0$. A cascading process will transport this energy to smaller and smaller scales until at the viscous length scale $\eta$ the injected energy is finally dissipated by viscous effects \cite{1, 2}. It is commonly believed that this picture of a cascade leads to the universality of the statistical laws of small scale turbulence.

The standard quantity to study the statistics of turbulent fields is the longitudinal velocity increment of the length scale $r$ defined as $v(r) = u(x + r) - u(x)$, where $u$ denotes the velocity component in the direction of the separation $r$. $x$ is a selected reference point.

To analyze the statistical content of the velocity increments two common ways have been established. On the one side, the structure functions have been evaluated by supposing a scaling behavior

$$< (v(r))^q > = \int dv \, (v)^q \, P(v, r) \propto r^{\zeta_q},$$

where $P(v, r)$ denotes the probability density function (pdf) for $v$ at the length scale $r$. Using the recently proposed so-called extended selfsimilarity \cite{3}, it has become possible to evaluate the characterizing scaling exponents $\zeta_q$ quite accurately c.f. \cite{4, 5}. On the other side, it is interesting to find out how to parameterize directly the evolution of the probability density functions (pdf) $P(v, r)$ c.f. \cite{6, 7}.

One major challenge of the research on turbulence is to understand small scale intermittency, which is manifested in a changing form of $P(v, r)$.
or equivalently in a nonlinear $q$-dependence of the scaling exponents $\zeta_q$, provided that the scaling assumption is valid.

For a long time, the main effort has been put into the understanding of the $q$-dependence of $\zeta_q$ (for actual reviews see [2, 5]), although it is well known that due to the statistics of a finite number of data points (let's say $10^7$ data) the determination of scaling exponents $\zeta_q$ for $q > 6$ becomes inaccurate [8]. In addition there are different experimental indications that no good scaling behavior is present [9, 10].

There have been less attempts to analyze directly the pdfs. This might be based on the fact that up to now the scaling exponents are regarded as the simplest reduction of the statistical content and that this analysis does not depend on model assumptions. In contrast to this, proposed parameterizations of the form of the pdfs [6, 7], although they are quite accurate, are still based on some additional assumptions on the underlying statistics. Based on the recent finding that the turbulent cascade obeys a Markov process in the variable $r$ and that intermittency is due to multiplicative noise [10, 11, 12], we show in Section III that it is possible to estimate from the experimental data a Fokker-Planck equation, which describes the evolution of the pdfs with $r$. We show that this Fokker-Planck equation reproduces accurately the experimental probability densities $P(v,r)$ within the inertial range (see Fig. 1). Thus, an analysis of experimental data which quantifies the statistical process of the turbulent cascade and which neither depends on scaling hypotheses nor on some fitting functions for pdfs is possible [13].

Having determined the correct Fokker-Planck equation for an experimental situation of local isotropic turbulence, we show in Section VI that with this approach also the statistics of inhomogeneous turbulence can be analyzed. Here, we present results for velocity data in a turbulent wake behind a cylinder. In two recent works it has been shown that the analysis of a Markov process in $r$ also holds for the energy dissipation averaged over distances $r$ [14, 15].

2. Experimental data

The results presented here are based on $10^7$ velocity data points measured in two different flows, namely, a free jet and a turbulent wake flow. Local velocity fluctuations were measured with a hot wire anemometer (Dantec Streamline 90N10). A single hot wire probe (55P01) with a spatial resolution of about 1 mm as well as an x-wire probe (55P71) with a spatial resolution of about 1.5 mm were used. The measurements were performed with sampling frequencies up to several kHz.

For the free jet, the stability was verified by measurements of the self-similar profiles of the mean velocity according to [16]. The turbulence mea-
measurements were performed by placing the probe on the axis of a free jet of dry air developing downwards in a closed chamber of the size of 2 m x 1 m x 1 m. To prevent a disturbing counterflow of the outflowing air, an outlet was placed at the bottom of the chamber. The distance of the hot wire to the nozzle was 125 nozzle diameters $D$. As a nozzle we used a convex inner profile [17] with an opening section of $D = 8$ mm and an area contraction ratio of 40. Together with a laminarizing prechamber we achieved a highly laminar flow coming out of the nozzle. At a distance of 0.25 $D$ from the nozzle, no deviation from a rectangular velocity profile was found within the detector resolution. Based on a 12bit A/D converter resolution, no fluctuations of the velocity could be found. For the experimental data used here, the velocity at the nozzle was 45.5 m/s corresponding to a Reynolds number of $2.7 \times 10^4$. At the distance of 125 $D$, we measured a mean velocity of 2.25 m/s, a degree of turbulence of 0.17, an integral length $L_0 = 67$ mm, a Taylor length $\theta = 6.6$ mm (determined according to [18]), a Kolmogorov length $\eta = 0.3$ mm, and a Taylor Reynolds number $R_\theta = 190$.

As a second experimental system, a wake flow was generated behind a circular cylinder inserted in a wind tunnel. Cylinders with two diameters $d$ of 2 cm and 5 cm were used. The wind tunnel [19] we used has the following parameters: cross section 1.6 m x 1.8 m; length of the measuring section 2 m; velocity 25 m/s; residual turbulence level below 0.1 %. To measure longitudinal and transversal components of the local velocity, the x-wire probes were placed at several distances between 8 and 100 diameters of the cylinder. Depending on the used cylinder, the Reynolds numbers based on the cylinder diameter were of $5 \times 10^4$ and $1.2 \times 10^5$. In the far field, the integral length varied between 10 cm and 30 cm, the Taylor lengths were around 2 mm, the Taylor Reynolds numbers were $R_\theta = 280$ and 650.

The space dependence of the velocity increments $v$ was obtained by the Taylor hypotheses of frozen turbulence. For the structure functions, we found a tendency to scaling behavior for $L_0 \geq r \geq \theta$. Intermittency clearly emerged as $r \to \theta$, as shown in Fig.1 by the different form of the pdfs for different scales $r$. Note that the velocity increments given in this paper are normalized to the saturation value of $\sigma_\infty = \sqrt{\langle v_\infty^2 \rangle}$ for length scales larger than the integral length.

3. Measurement of Kramers-Moyal coefficients

Next, we show how to determine from experimental data appropriate statistical equations to characterize the turbulent cascade. The basic quantity for this procedure is to evaluate the cascade by the statistical dependence of velocity increments of different length scales at the same location $x$ [20]. Either two-increment probabilities $p(v_2, r_2; v_1, r_1)$ or corresponding condi-