Assessment of turbulence by high-order statistics. Offshore example.

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1 Summary

Offshore turbulent statistical features are analyzed for two different sites by means of ultrasonic anemometry. Traditionally only turbulence intensity and spectral features are studied for wind energy applications. Besides these statistical quantities we analyzed so called two-point higher order statistics for data sets conditioned on a 10-minute mean wind speed. We contrast our results with homogeneous isotropic turbulence theory (Kolmogorov 62). We show that the turbulence intensity seems to follow Log-Normal probability density functions. We also show that statistical structure corresponding to homogeneous isotropic turbulence is clearly found once the effect of different turbulence intensity is filtered away.
Keywords: Offshore turbulence. Intermittency. Turbulence Intensity. Log-Normal distributions.

2 Data sets

We use two Data sets in our analysis. The main details of those data sets are summarized in Table 1. The first data set comes from the well known research platform FINO I in the North Sea. The second one is coming from the Östergarnsholm field station in the Baltic Sea. This station is in an island but it has been shown [1] that at specific directions the measurements correspond to oceanic conditions. Both datas sets are meausured with Gill ultrasonic anemometers.

2.1 Quality Control

In the case of FINO I a data set with directions coming from the NorthWest direction was selected. This assures low disturbance in the measurements due to the wake of the mast. Additionally a quality control was carried on the data. Namely a despiking of isolated events corresponding to deviations bigger than 5σ from a ten minute wind speed average was done. In the case of the Östergarnsholm station no despiking was applied. Overall the data quality of this latter station is considered high and widely documented (e.g. [1]).

Table 1: Data basic characteristics.

Sources	*FINO I*	*Östergarnsholm*
f[Hz]	10	20
Mean Speed $[ms^{-1}]$	11	5.4
Height $[m]$	80	25
Number of points	11954270 (14Days)	36,888,000 (21 Days)

3 Method

3.1 Stationarity

In this paper we are not interested in the effects in turbulence due to the variation of the mean wind speed. Therefore we conditioned our data on mean wind speed before we estimate any turbulence measurement. This should ensure certain stationarity in our data. In particular we choose FINO I data sets that fulfill the condition $\bar{u} = 10ms^{-1} \pm 1ms^{-1}$. In the case of Östergarnsholm data we use data sets that fulfill the condition $\bar{u} = 6ms^{-1} \pm 1ms^{-1}$.

3.2 Turbulence Statistics

We will estimate fluctuations of the along wind component of the velocity vector. In particular:

$$u(t)' = u(t) - \bar{u}_T, \quad (1)$$

where the direction of u is parallel to the 10-minute mean velocity vector[1]. As presented in [2]

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[1]Consider the horizontal components of the velocity vector, $\vec{u} = \hat{i}u + \hat{j}v$. The reference system of the ultrasonic anemometers was rotated such that the direction of $\hat{i}$ is align with the 10

when analyzing turbulent time series one should, at least in principle, consider a hierarchical statistical structure. This is particular important since turbulence is a complex phenomena with different statistical features unfolding different physical features and one should choose the right statistical quantity for the desired application. In this work we will study turbulence intensity (one-point second order statistics), power spectrum density (two-point second order statistics) and the intermittency or shape factor which summarizes (two-point high order statistics). For clarity we define briefly the afore mentioned statistical quantities: We will estimate the well known turbulence intensity

$$TI = \frac{\sigma(u')_T}{\bar{u}_T} = \frac{\sqrt{\overline{u'^2}}}{\bar{u}_T}. \quad (2)$$

Where $\bar{x}_T$ is the mean of x over the time span T. In this paper $T = 10min$. We will also estimate the power spectrum density $S(f)$, as the Fourier transformation of the autocorrelation function.

$$R(\tau)_{u'u'} = \frac{\overline{u'(t+\tau)u'(t))}}{\sigma^2(u')_T} \quad (3)$$

In particular we recall that according to Kolmogorov in the case of homogeneous isotropic turbulence (HIT) the power spectrum should decay with frequency as:

$$S(f) \propto f^{-5/3}. \quad (4)$$

Finally and crucial for this work. We will analyze the so called shape parameter λ_τ^2 of wind speed differences $\delta u'(t)_\tau = u'(t+\tau) - u'(t)$ defined like:

$$\lambda_\tau^2 = \frac{log(F_\tau/3)}{4} \quad (5)$$

Where $F_\tau = \frac{\overline{(\delta u'(t)_\tau)^4}}{\overline{(\delta u'(t)_\tau)^2}^2}$, some times call flatness. For a Gaussian distributed random variable $F = 3$. Therefore $\lambda_\tau^2 = 0$ implies a Gaussian distribution for wind speed increments $\delta(t)_\tau$. Details on the meaning of λ_τ^2 and its role in the statistical theory of HIT can be found in [2]. Here we only recall that the greater the value of λ_τ^2 the greater the probability of extreme values of wind speed differences $u'(t+\tau) - u'(t)$. And again according to Kolmogorov the decay of the shape factor with scale should be like:

$$\lambda_\tau^2 = -\mu \ln \tau + \Lambda^2. \quad (6)$$

In this paper we will estimate λ_τ^2 and look for a region of time scales where a law of the form of eq. (5) holds. If we find such a region, that suggests strong indications of turbulence behaving according to the expectations of HIT.

minute mean direction. In this rotated system v is naturally averaging to zero.

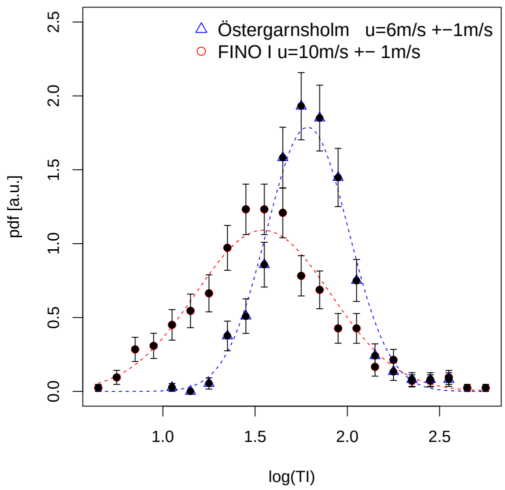

Figure 1: Probability density function of the logarithm of the turbulence intensity (TI). Symbols represent measurements and lines are Gaussian distributions with the same mean and standard deviations as the respective data.

4 Results

4.1 Turbulence Intensity

Figure 1 shows the probability density function (PDF) of the measured turbulence intensities (of $log(TI)$) for conditioned data on mean wind speed of 10m/s and 6m/s for FINO and Östergarnsholm respectively. According to [3] this PDFs are closely Log-Normal. This seems to be the case for our data as well. Supporting the Log-Normal hypothesis, in the mentioned figure equivalent Normal distributions with the same mean and standard deviation as $log(TI)$ are shown as dotted lines . Moreover, in Fig. 2, we show the normal score plot of measured (points) and the fitted (line) cumulative distribution function (CDF). Details on the normal score plot method can be found in [3] . If the PDF of turbulence intensities is log-Normal then, a nice linear fit should be possible. Besides some outliers the normal score plot method confirms the Log-Normal distribution of turbulence intensities. On the the other hand the difference in both PDFs probably reflect that the sets are taken at different sites, different heights and with a different combination of thermal stratifications.

4.2 Power Spectrum Density

In fig. (3) we show the estimated power spectral density for both sites. The calculation has been done independently every 10-min. and then an

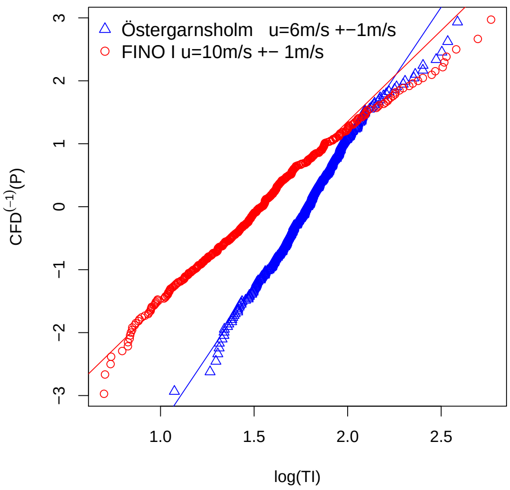

Figure 2: Normal score plot of measured (points) and fitted (line) cumulative distribution function (CDF). If the PDF of turbulence intensities is log-Normal then, a nice linear fit should be possible.

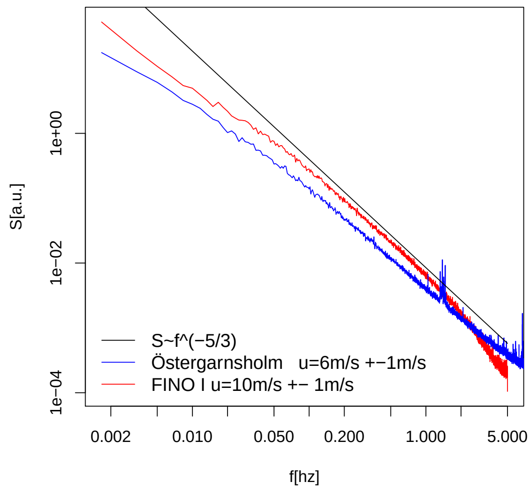

Figure 3: Power spectral density for the two offshore data sets. The calculation has been done independently every 10-min. and then an average of the 10-min. spectra has been done. The area under the curve is proportional to the variance of the whole time series of u'.

average of the 10-min. spectra has been done. The area under the curve is proportional to the variance of the whole time series of u'. For both sites the spectral density seems to follow power law similar to eq. (4). This gives the indication that we are working with fully developed turbulence in this frequency region.

4.3 Two-point higher order statistics

Now we focus in the statistics of wind speed increments. We estimate λ^2_τ according to Eq. (5). The range of time scales τ starts at the smallest time step available ($0.1s$ for FINO I and $0.05s$ for Östergarnsholm station) and we go up to almost $600s$. Note that we estimate the statistics of wind speed increments separately for every 10-min window without overlapping windows. That means that for the biggest time scales we have considerably less statistics. In Fig. 4 we show the results of the calculation for both sets. We also show as a dotted line the expectations of HIT theory according to Eq. (6). In the case of Östergarnsholm station, a clear region of scales seem to be behaving as HIT. However for the smallest as well as for the the biggest time scales clear deviations are seen. In contrast to this for FINO I data the scaling of λ^2_τ with the time scale τ do not follow the expectations of Kolmogorov theory at all. Differences between the two sites are not surprising since both data sets have different local conditions partly reflected on the results of the previous section. However we would have expected that at least at small scales turbulence should exhibit some universal features despite the different origin. In an attempt to unfold better universal features of turbulence, we refined our analysis filtering out the effect of the different turbulent intensities. Note that the turbulence intensity is only a measurement of the amplitude of the turbulent fluctuations from the mean. And by itself do not tells us anything about the physical process or structure follow by the turbulent flow (wind). In other words the turbulence intensity only tell us how "strong" turbulence is, but do not tell us "how" it behaves. In order to carry the filtering of turbulence intensities we now estimate λ^2_τ with the statistics of $\tilde{u} = \frac{u'}{\sigma(u')_T}$ rather than with u'. Where the normalization by $\sigma(u')_T$ is done every ten minutes. The results of these normalized statistics are shown in fig. 5. It is astonishing how well the shape parameter λ^2_τ now almost perfectly follows Eq. (6) and this for both data sets at almost all time scales. We will elaborate more on the meaning of this result in the conclusions.

5 Conclusion

We have analyzed basic turbulent characteristics of two offshore sites based on ultrasonic anemometry. It has been found that in line with published research the turbulence intensities for given mean wind speed bins seem to follow Log-Normal probability density distributions. The power spectral

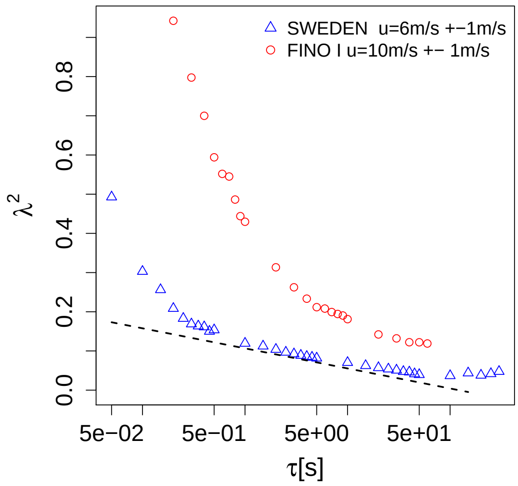

Figure 4: $\lambda_\tau^2 = \frac{log(F_\tau/3)}{4}$ vs $\tau[s]$. Where $F_\tau = \frac{\overline{(\delta u'(t)_\tau)^4}}{\overline{(\delta u'(t)_\tau)^2}^2}$, $\delta u'(t)_\tau = u'(t+\tau) - u'(t)$. The dotted line corresponds to the expectations of Kolmogorov 62 theory for HIT according (see eq. (6)).

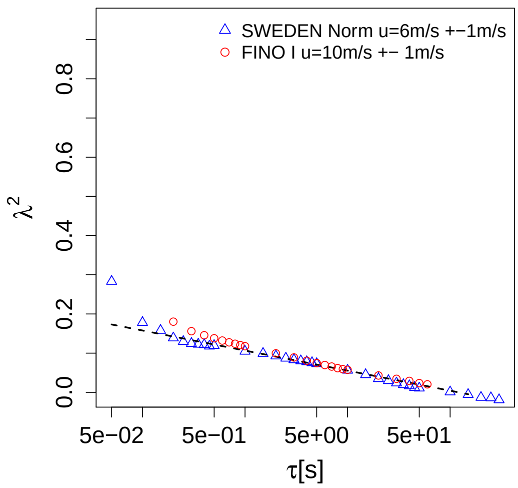

Figure 5: Similar to Fig.(4). $\lambda_\tau^2 = \frac{log(F_\tau/3)}{4}$ was estimated using increments of $\tilde{u} = \frac{u'}{\sigma(u')_T}$, such that $\delta\tilde{u}'(t)_\tau = \frac{u'(t+\tau)-u'(t)}{\sigma_T}$. Note that the value of σ_T changes every 10-minutes and is proportional to the turbulence intensity.

density properties of the wind time series show a clear region were an inertial range is suggested. However when two-point higher order statistics are analyzed, clear deviations from homogeneous isotropic turbulence are observed for FINO I and at some scales for Östergarnsholm station. We have shown that the deviations from HIT can be accounted as coming from the different distributions of turbulent intensities in the sites. This result is quite encouraging since it gives robust indications that at least at fast temporal scales (less that 10-min), the structure of offshore turbulence can be treated in a simplified way following HIT hypothesis. The only prerequisite is the filtering of the additional randomness of the turbulence intensity at the given mean wind speed bin. In this sense, the variation of the turbulent intensity for each mean wind speed as expected seems to hold important information related to particular site conditions. Moreover, the decay shown in fig. (5) enables, at least in principle, the possibility for inffering spatial information of turbulent structures using one-point measurements. This since a decay of $\lambda_\tau^2 \propto \mu ln(\tau)$ would be extremely difficult to reach if isotropic and homogeneous properties were not satisfied in the field. We have checked the value of μ and it is consistent with wind tunnel experiments where homogenity and isotropy is better fulfilled. In future work we will show how to use the findings in this work in order to estimate probabilities of wind speed differences in space and time.

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References

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